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Attacks on the Firekite Cipher

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  - Alternatively, learning assumptions such as LPN, LWE.
- Learning Parity with noise is appealing in many applications for its simplicity.

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- Not suitable for low-weight, restrained devices.

The Firekite cipher (Bogos et al.)

Design [BKLV21]

Assume $n \gg m$, and k are integers.

$$\overset{m}{\mathbf{v}} \cdot \overset{m \times n}{\mathbf{M}} + \underset{\uparrow \atop \text{error vector of weight } k}{\overset{n}{\mathbf{e}}} = \begin{pmatrix} \mathbf{g} \\ \mathbf{v}' \\ \mathbf{c}_e \end{pmatrix}$$

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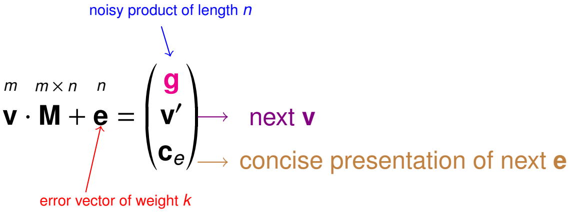

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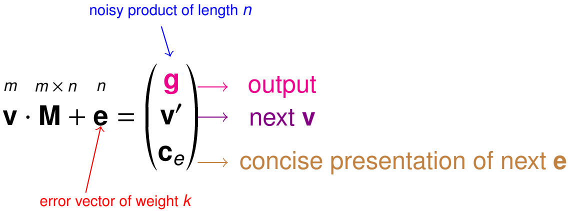

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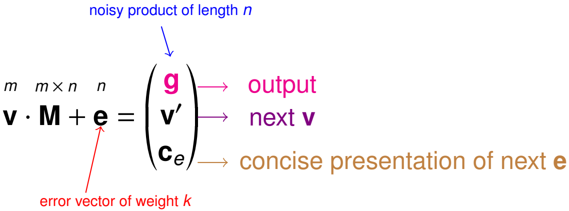

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- Keystream length is $d = n - m - k \cdot \log n$.
- For efficiency, the authors proposed using a 'cyclic' **M**.

The Learning Parity with Noise Problem

LPN oracle.

Let $\mathbf{x} \xleftarrow{\$} \{0,1\}^m$ and $\eta \in (0, \frac{1}{2})$. An LPN *oracle* Π_{LPN} for $\mathbf{x}$ and η returns pairs of the form

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where $e \leftarrow \mathrm{Ber}_\eta$, and $\langle \mathbf{x}, \mathbf{g} \rangle$ denotes the scalar product of vectors $\mathbf{x}$ and $\mathbf{g}$.

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LPN problem, Search version, informal.

Given an LPN oracle Π_{LPN} with parameters m and η. The (m, η)-LPN problem is finding the secret vector $\mathbf{x}$ from observing N samples from (m, η)-Π_{LPN} oracle.

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Remark:

It is closely related to the *Syndrome Decoding Problem.*

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Security:

- Reduction to LPN.
- Apply cryptanalysis methods to the (corresponding) LPN-instance.

Assumption:

- Ring-LPN is secure.
- A Firekite instance is as hard as its (corresponding) LPN-instance.

Distinguishing Attacks

Key Observations

$$\mathbf{v}_i \cdot \mathbf{M}_{[d]} + \mathbf{e}_i = \mathbf{g}_i.$$

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- If $\sum_{i=1}^{\ell} \mathbf{v}_i = \mathbf{0}$, then $\sum_{i=1}^{\ell} \mathbf{g}_i = \sum_{i=1}^{\ell} \mathbf{e}_i$. Moreover, $\mathbf{e}_i$ is sparse ($k \ll n$).

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- Since $m < d$, we expect to see low Hamming-weight combinations more frequently than the random case.

How to exploit the observations?

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Figure: Applying our ideas with observing the keystream.

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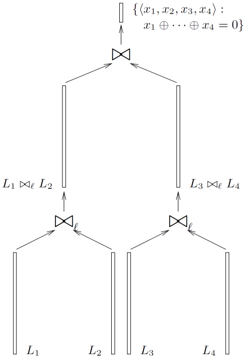
Idea:

If we can detect a low-weight sum of $\mathbf{g}_i$, and it is statistically implausible to have such a sum in random case, then it must have come from a collision in $\mathbf{v}_i$.

How to efficiently detect low-weight sums?



A pictorial representation of our algorithm for the 4-sum problem.

Figure: Match-and-Filter [BM17]

Figure: Wagner algorithm [Wag02]

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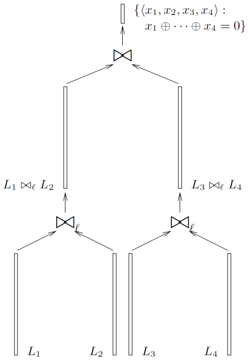

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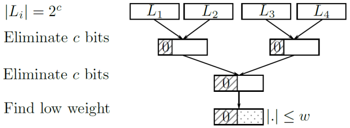

Match-and-Filter algorithm for $k = 4$

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Our algorithm

We apply essential ideas and arguments from the two above algorithms, with some flavor from BKW-algorithm to detect low-weight ℓ-sums.

Modifications for our algorithm

- Instead of ℓ lists, we use only 1 initial list $L^{(0)}$, with an increased size. In particular, to cancel c bits and maintain the list size, $L^{(0)} \approx 3 \cdot 2^{c}$.

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- We call COMBINE the routine to find vectors that collide in c bits. Let $t = \log \ell$, we need to apply COMBINE t times, resulting in $L^{(0)} \longrightarrow L^{(1)} \cdots \longrightarrow L^{(t)}$.
- The parameter c in our algorithms needs to be bigger than in Wagner's algorithm. The reason is, we also need the observed g_i to be at least *error-free* modulo 2 in $t \cdot c$ positions.

Our algorithm, Combine.

- We can use *c* tuples as indices/keys in a hash table and detect collisions in each iteration.

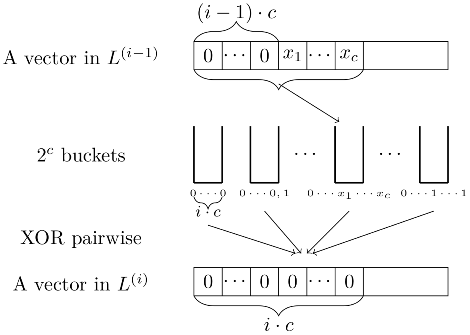
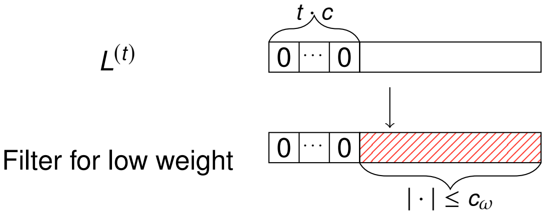


Figure 1: Combine for $L^{(i-1)}$.

Our algorithm, Filter



$L^{(t)}$

Filter for low weight

Figure: Filter $L^{(t)}$ with c_ω.

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- A: Assume P_{nf} is defined as the probability a low-weight sum is error-free modulo 2 in the first tc bits. If Wagner algorithm requires c, we need an overhead $\alpha(P_{nf})$, so $c + \alpha(P_{nf})$.

Analysis

Memory (the exponent c).

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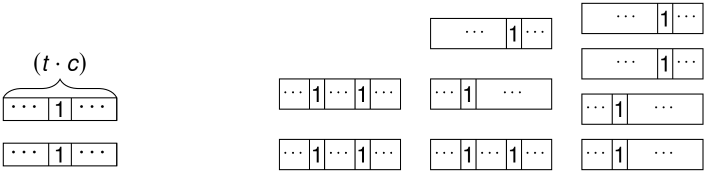
Remark

- The better we ‘estimate’ P_{nf}, the smaller $\alpha(P_{nf})$ is.
- For $\ell = 8$, we consider more complicated error patterns in the first $t \cdot c$ bits.

Some examples of the error colliding patterns in canceled bits.



(a) 1 double error

(b) 2 double errors

Figure: Illustration for the colliding patterns

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$$N = \binom{3 \cdot 2^c}{8} \cdot 2^{-m} \cdot 105 \cdot 2^{-4c} \cdot P_{\mathrm{nf}} > 1$$

Complexity

$$C = t \cdot (3 \cdot 2^c) \cdot (1 + \lfloor d/p \rfloor).$$

On average, we have to do $3 \cdot 2^c$ XOR operations in each iteration of Combine. Each XOR cost $1 + \lfloor d/p \rfloor$, where p is the number of bits that can be XOR-ed in each operation.

Note

Of course, there are other algorithmic costs but this is the dominating part.

Success Probability

How to ‘interpret’ the low-weight sums that have been found?

The low-weight sums must be easily distinguished from those that can happen by sheer chances. In other words, it must be statistically improbable for such a low-weight sum to appear.

$$N_{\text{random}} = 3 \cdot 2^{c} \cdot \frac{\sum_{i=0}^{c_\omega} \binom{d-t\cdot c}{i}}{2^{d-t\cdot c}} \approx \sum_{i=0}^{c_\omega} 2^{-\left(1-H\left(\frac{i}{d-t\cdot c}\right)\right)(d-t\cdot c)+c} \approx 2^{-\left(1-H\left(\frac{c_\omega}{d-t\cdot c}\right)\right)(d-t\cdot c)+c}.$$

Results

Attacks on Firekite with different parameters.

Table: Our distinguishing attack complexity for 80-bit and 128-bit security of Firekite.

Parameters				Memory (c)		Time(log)		N_{random}(log)	
m	n	k	Security	4-sum	8-sum	4-sum	8-sum	4-sum	8-sum
216	1024	16	82.76	76	62	80.17	66.75	-215.76	-90.23
216	2048	32	82.76	76	62	81.17	67.75	-765.79	-465.74
216	16,384	216	80.68	75	60	83.28	68.87	-9011.62	-6541.71
352	2048	32	129.07	125	101	130.16	106.75	-541.40	-275.94
352	4096	58	128.95	124	99	130.17	105.75	-1739.41	-1150.69
352	16,384	228	128.93	123	99	131.26	107.84	-8510.19	-6023.39

Key Recovery (in prose)

How does ISD algorithm work?

Problem:

Let C be a random binary code generated by a matrix $\mathbf{G} \in \mathbb{F}_2^{k \times n}$. Let $\mathbf{y} = \mathbf{c} + \mathbf{e}$ be a noisy codeword where $\mathbf{c} \in C$, and $\omega_H(\mathbf{e}) = \omega <$ minimum distance of C. Recover $\mathbf{y}$, or $\mathbf{e}$.

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$$\begin{pmatrix} \mathbf{G} \\ \mathbf{y} \end{pmatrix} \xrightarrow{Gauss \circ \pi} \begin{pmatrix} \mathbf{I} & \mathbf{J} \end{pmatrix}$$

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Let C be a random binary code generated by a matrix $\mathbf{G} \in \mathbb{F}_2^{k \times n}$. Let $\mathbf{y} = \mathbf{c} + \mathbf{e}$ be a noisy codeword where $\mathbf{c} \in C$, and $\omega_H(\mathbf{e}) = \omega <$ minimum distance of C. Recover $\mathbf{y}$, or $\mathbf{e}$.

$$C' = C + \mathbf{y} = \langle \begin{pmatrix} \mathbf{G} \\ \mathbf{y} \end{pmatrix} \rangle.$$

Equivalently, we find a codeword of weight ω of this new code.

$$\begin{pmatrix} \mathbf{G} \\ \mathbf{y} \end{pmatrix} \xrightarrow{Gauss \circ \pi} \begin{pmatrix} \mathbf{I} & \mathbf{J} \end{pmatrix}$$

Assume the first part is of dimension k, we run through weight p vectors $\mathbf{u}$ of length k and check for $\omega_H(\mathbf{uJ}) = \omega - p$.

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- Estimate the double errors, then apply ISD algorithms.

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- Further works.

References

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